

the values of ϵ for the long slit are lower than the values for the circular orifice. However the long slit may be more practical because of the reduced equipment cost; for example a circular orifice 1 mm. in diameter will pass only 700 g. uranium hexafluoride/hr. at a pressure of 30 mm. Hg and a velocity of 1,000 ft./sec., while a slit 1 mm. by 1 m. will pass 900 kg. under the same conditions. Thus the smaller enrichment may be justified by the greatly increased throughput made possible by the long slit.

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NOTATION

- A = separation coefficient for a two-component mixture
 f = fraction of feed entering the side stream
 k = gas law constant
 m = mass of molecule
 n_o = total number of molecules
 T = absolute temperature
 V = molecular velocity
 V_{ox} = bulk-gas velocity at orifice
 V_x = molecular velocity component in the x direction
 V_y = molecular velocity component in the y direction
 V_z = molecular velocity component in the z direction
 $V.$ = sum of x and y velocity components

Greek Letters

- α = angle between $V.$ and V_y
 β = dimensionless parameter, $\beta =$

$$V_{ox} \sqrt{\frac{m}{2kT}}$$

- β_u = value of β for the member of a two-component mixture having the higher molecular weight
 β_L = value of β for the member of a two-component mixture having the lower molecular weight
 ϵ = enrichment factor for a two-component mixture
 θ = angle between V and the x axis
 ϕ = angle at which jet is broken into two streams by the divider

LITERATURE CITED

1. Becker, Bier, and Buyhoff, *Z. Naturforsch.*, 10a, 565 (1955).
2. Becker, *ibid.*, 12a, 609 (1957).

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COMMUNICATIONS TO THE EDITOR

Dear Editor:

The policy of publishing translations of the better papers which appear in Japanese Journals is a good one. Since we have been studying heat transfer to Bingham plastics, the paper by Hirai which appeared recently in the *Journal* (1) was not unknown to us; however having the English translation was quite helpful. The problem of heat transfer to a Bingham plastic in laminar flow is important because of the extreme velocities required to attain turbulent flow, and consequently in many industrial applications the flow will be laminar. The same problem considered by Hirai has been solved independently by the writers (2) using an entirely different approach. Owing to differences in the mean temperature reported by Hirai in Figure 4 of the original paper and the mean temperatures that we calculated, we have studied the paper rather carefully. The following errors or omissions were noted.

Equations (1.18), (1.19), and (1.20) are improper because the functions $P_m(\xi)$ are not orthogonal on the interval $(a, 1)$. This can be seen by observing that $P_m(\xi)$ satisfies

$$-\frac{d^2 P_m}{d\xi^2} + \frac{1}{\xi} \frac{dP_m}{d\xi} + \beta_m^2 \left[1 - \right.$$

$$\left. \left(\frac{\xi-a}{1-a} \right)^2 \right] P_m = 0 \quad (1.10)$$

$$P_m(1) = 0$$

$$\frac{dP_m(0)}{d\xi} = 0 \quad (\text{not specified by Hirai})$$

Equations (1.18), (1.19), and (1.20) are valid only if

$$\int_a^1 P_m(\xi) P_n(\xi) \xi \left[1 - \left(\frac{\xi-a}{1-a} \right)^2 \right] d\xi$$

$$= 0 \quad \text{if } m \neq n$$

$$= \text{constant} \quad \text{if } m = n$$

From Equation (1.10)

$$\int_a^1 P_m(\xi) P_n(\xi) \xi \left[1 - \left(\frac{\xi-a}{1-a} \right)^2 \right] d\xi$$

$$= \left(\frac{a}{\beta_m^2 - \beta_n^2} \right) \left[\frac{dP_m(a)}{d\xi} P_n(a) - \frac{dP_n(a)}{d\xi} P_m(a) \right]$$

The right-hand side can be zero only if

$$P_m(a) = 0$$

or if

$$\frac{1}{P_m(a)} \frac{dP_m(a)}{d\xi} = \frac{1}{P_n(a)} \frac{dP_n(a)}{d\xi}$$

Referring to Figure 3 of Hirai's original

paper, we see that neither of these conditions is satisfied by P_0 , P_1 , and P_2 . Consequently the functions $P_m(\xi)$ are not orthogonal, and even if a set of A_m 's existed as defined in Equation (1.18), they could not be determined by the use of Equations (1.19) and (1.20).

We wonder why Hirai has not mentioned the second solution of Equation (1.10). This solution cannot be thrown out because of its singularity at $\xi = 0$, since the solution is used only in the region from $\xi = a$ to $\xi = 1$. As a result of this omission Hirai has apparently satisfied only the condition

$$\theta(r_p) = \theta_p(r_p)$$

and assumed that the condition

$$\frac{\partial \theta(r_p)}{\partial r} = \frac{\partial \theta_p(r_p)}{\partial r}$$

is satisfied.

1. Hirai, E., *A.I.Ch.E. Journal*, 5, 13 (1959).
2. Wissler, E. H., and R. S. Schechter, *Chem. Engr. Progr. Symposium Ser. No. 29*, 55, 203 (1959).

Yours very truly,

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